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HEAT AND MASS TRANSFER IN THE HETEROGENEOUS SYSTEM MATRIX–INCLUSIONS UNDER PULSE ELECTRON-BEAM PROCESSING

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A mathematical model of pulse electron-beam processing of a heterogeneous material with allowance for the fact that each phase (particles and the matrix) possess its own properties and thus is characterized by its own concentration and temperature has been proposed. The processes of transfer of heat and mass in particles and the matrix and exchange processes between the phases have been taken into account. An example of a partial problem enabling one to calculate the coefficients of heat and mass exchange between the phases as functions of the temperature and the particle size has been given. The influence of the character of the external heat flux on the temperature and concentration fields in particles and the matrix has been illustrated.

Introduction. Surface treatment of materials with external energy sources that leads to a modification of their properties finds ever-increasing use in the strengthening of cutting tools, deposition and modification of coatings on the surface of wearable parts, formation of gradient transition layers between coatings and the base, etc. A number of fundamental results obtained in the 1960–80s are contained in monographs and in the reference literature. Unfortunately, mathematical modeling in this field lags behind the needs of practice. Thus, the existing mathematical models of the processes of surface treatment are reduced to purely thermophysical problems in which the temperature dependence of the effective thermophysical properties and the processes of melting and crystallization are allowed for at best [1, 2]. Also, we know of publications in which the residual stresses in the parts treated are calculated from the known temperature field formed in the process of treatment of material with effective properties. Simplified representations often turn out to be sufficient in modeling surface treatment of metals. Electron-beam processing of heterogeneous materials have distinctive features related to the different character of heat and mass transfer in different phases (e.g., in particles and the matrix) and to the character of their interaction dependent on the specific chemical composition, shape, and size of particles, conditions of wettability of their surface by the molten base, etc. Thus, it is well known that a composite material consisting of tungsten carbide (WC) particles embedded in a cobalt binder is inconvenient in practical use because of the poor contact between the particles and the matrix. Flaking of particles in the process of operation leads to fast wear of parts. It has experimentally been shown in [3] that the electron-beam processing of WC in the cobalt binder in the regime of single pulses of 10^{-4} sec and an energy density in the beam of ≥ 100 J/cm^2 causes the carbon in WC to diffuse into Co and is accompanied by the local melting of the sample in the vicinity of the internal boundaries. It has experimentally been confirmed [3–6] that in electron-beam processing of a heterogeneous material or in electron-beam deposition of particles on the base by welding, the high-melting particles in the molten liquid-metal bath of the base accelerate the process of its crystallization and lead to a decrease in the residual stresses in the sample's surface layer or in the deposited (surfaced) coating. In this case the so-called solid-solution strengthening (formation of a solid solution of increased strength) is realized in the process of exposure due to the dissolution of alloy components (Mo, N, and C).

The high-energy electron-beam pulse method is efficient for surface treatment of instruments manufactured from different materials (steels, solid alloys, and coatings applied by spraying) with the aim of increasing their service life [7]. In treating the composite alloy TiC + NiCr characterized by a good contact between particles and the matrix (or a good wettability of the particles surface with the molten matrix), strengthening is realized due to the dissolution of a certain number of particles because of the mutual diffusion of the components (titanium and carbon from the par-

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ticles into the matrix and nickel and chromium from the matrix to the particles) and, as a consequence, the size reduction in the structure of the heterogeneous material.

The above features of heterogeneous materials disregarded in all the existing theoretical works lead to both the need for special experimental investigation and a formulation of the corresponding mathematical models. On condition that the particle size is small compared to the width of the warming-up zone formed in the process of treatment, the correct model of electron-beam processing of a heterogeneous material with effective thermophysical properties has been described in [8]. For arbitrarily sized particles we must explicitly allow for the complex character of heat and mass exchange between particles and the matrix [9, 10].

To illustrate the role of interphase interaction we consider below the model of heat and mass transfer in a heterogeneous material in pulse electron-beam processing, which allows for different velocities of the transfer processes in particles and the matrix and for interphase exchange.

Formulation of the Problem. We consider the model of electron-beam processing of a heterogeneous material in a two-temperature approximation. Each phase will be characterized by its own concentrations C_i and temperature T_i (i = 1 and 2). In the general case there can be exchange of mass, heat, and momentum between the phases; chemical compounds can be formed in each phase. In the simplest case we restrict ourselves to an analysis of the mass transfer of one component responsible for the formation of transition layers between particles and the matrix (e.g., carbon or titanium in the case of a composite material based on the nickel-chromium binder of titanium-carbide particles). The thermal expansion of the composite as a result of heating and the resulting stresses in the first approximation are disregarded.

A mathematical formulation of the problem involves two heat-conduction equations and two diffusion equations for particles and the matrix respectively

$$(c_1\rho_1)_{\text{eff}} \frac{\partial T_1}{\partial t} = \frac{\partial}{\partial x}\left(k_1 \frac{\partial T_1}{\partial x}\right) - \frac{\alpha_{12}}{\eta}(T_1 - T_2) \ , \tag{1}$$

$$(c_2\rho_2)_{\text{eff}} \frac{\partial T_2}{\partial t} = \frac{\partial}{\partial x}\left(k_2 \frac{\partial T_2}{\partial x}\right) + \frac{\alpha_{12}}{1-\eta}(T_1 - T_2) + \sum_j Q_j w_j \ , \tag{2}$$

$$\frac{\partial C_1}{\partial t} = \frac{\partial}{\partial x}\left(D_1 \frac{\partial C_1}{\partial x}\right) - \frac{\beta_{12}}{\eta}(C_1 - C_2) \ , \tag{3}$$

$$\frac{\partial C_2}{\partial t} = \frac{\partial}{\partial x}\left(D_2 \frac{\partial C_2}{\partial x}\right) + \frac{\beta_{12}}{1-\eta}(C_1 - C_2) + \sum_j w_j \ . \tag{4}$$

The diffusion coefficients of the alloy component in particles and in the matrix are temperature-dependent according to the Arrhenius law

$$D_1 = D_{10} \exp\left(-\frac{E_{\text{a1}}}{RT_1}\right), \quad D_2 = D_{20} \exp\left(-\frac{E_{\text{a2}}}{RT_2}\right).$$

The effective heat of the matrix sharply grows in the vicinity of the melting temperature T_{ph2}, which is reflected by the dependence

$$(c_2\rho_2)_{\text{eff}} = Q_{\text{ph2}}\rho_{2\text{s}}\delta\,(T - T_{\text{ph}}) \begin{cases} (c_2\rho_2)_{\text{s}} \ , & T < T_{\text{ph2}} \ ; \\ (c_2\rho_2)_{\text{liq}} \ , & T > T_{\text{ph2}} \ , \end{cases} \qquad \delta = \begin{cases} 0 \ , & T \neq T_{\text{ph2}} \ ; \\ \infty \ , & T = T_{\text{ph2}} \ . \end{cases} \tag{5}$$

An analogous formula generally holds for particles.

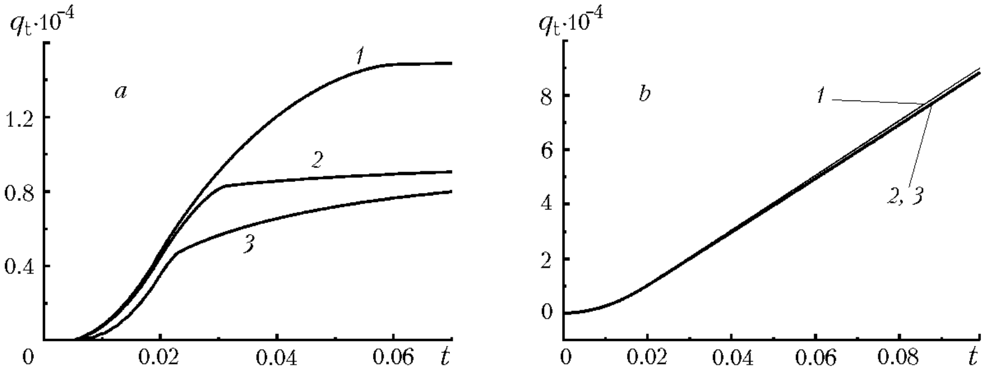

Fig. 1. Heat storage vs. time calculated from formula (10) based on the results of numerical calculation for a constant external flux and different values of the heat-exchange coefficient [1) α_{12} = 1, 2) 100, and 3) 10,000 W/(cm^2·K); q_0 = 2500 W/cm^2]: a) width of the warming-up zone X_t is determined from the temperature value max (T_1, T_2) > 700 K; b) width of the warming-up zone $X_t \to \infty$. q_t, J/cm^2; t, sec.

The last terms in Eqs. (2) and (4) characterize possible heat and mass sources (sinks) because of the chemical reactions. Under surface pulse electron-beam processing, on the surface $x = 0$, we have

$$-k_1 \frac{\partial T_1}{\partial x} = -\sigma\varepsilon_1 (T^4 - T_{\text{env}}^4) + \eta q(t), \quad -k_2 \frac{\partial T_2}{\partial x} = -\sigma\varepsilon_2 (T^4 - T_{\text{env}}^4) + (1 - \eta)\, q(t); \tag{6}$$

$$\frac{\partial C_1}{\partial x} = 0, \quad \frac{\partial C_2}{\partial x} = 0, \tag{7}$$

where

$$q(t) = q_0 f(t); \quad f(t) = \sum_{k=1}^{n} f_k(t); \quad f_k(t) = \begin{cases} 1, & (t_{\text{imp}} + t_{\text{p}})(k-1) \le t < t_{\text{imp}} + (t_{\text{imp}} + t_{\text{p}})(k-1); \\ 0, & t_{\text{imp}} + (t_{\text{imp}} + t_{\text{p}})(k-1) \le t < (t_{\text{imp}} + t_{\text{p}})\, k; \end{cases}$$

n is the number of pulses.

At infinite distance from the surface treated, heat and mass sources and sinks are absent

$$x \to \infty: \quad \frac{\partial T_1}{\partial x} = 0, \quad \frac{\partial T_2}{\partial x} = 0; \quad \frac{\partial C_1}{\partial x} = 0, \quad \frac{\partial C_2}{\partial x} = 0. \tag{8}$$

At the initial instant of time, we specify the condition

$$t = 0: \quad T_1 = T_2 = T_0; \quad C_1 = C_{10}; \quad C_2 = 0. \tag{9}$$

It is necessary to investigate the behavior of temperature and concentrations under different combinations of the number of pulses, pulse durations, and pauses for different conditions of interphase heat and mass exchange and to theoretically consider the regimes of treatment of the material surface for further practical study of the model.

Thermophysical Model. Problem (1)–(9) was solved numerically; the differential equations involved in the model were approximated by the implicit difference scheme by the four-point template. The running method was used to solve the resulting system of linear algebraic equations. The Dirac delta function was replaced by a delta function satisfying the normalization condition which reflected the conservation of internal energy in the processes of melting and crystallization.

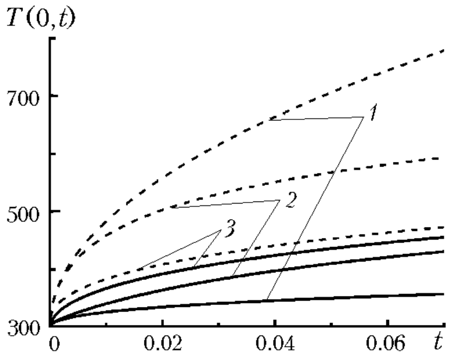

Fig. 2. Surface temperature of particles (dashed curves) and the matrix (solid curves) vs. time for a constant external flux and different values of the heat-exchange coefficient: 1) α_{12} = 1, 2) 100, and 3) 10,000 W/(cm^2·K); q_0 = 2500 W/cm^2. t, sec.

In the calculations, we determined the temperature and concentration fields for particles and the matrix at different instants of time and the width of the warming-up zone X_t (from the temperature value max (T_1, T_2) ≥ 700 K) and the heat storage in the system q_t as a result of the external heating:

$$q_t = \int\limits_0^{X_t} [c_1\rho_1 (T_1 - T_0)\, \eta + c_2\rho_2 (T_2 - T_0)(1 - \eta)]\, dx\,. \tag{10}$$

Numerical values of all quantities are substantially dependent on the model's parameters. If the properties of particles and the matrix are different, the width of the warming-up zone and the heat storage determined in accordance with formula (10) are dependent on the value of the coefficient of interphase heat exchange (Fig. 1a).

The quantity of heat flowing from the particles to the matrix grows with α_{12}, which leads to a reduction in the particle temperature and an increase in the matrix temperature; as a result the temperature of the entire sample decreases, which leads to a contraction of the warming-up zone (not shown in the figure) and hence to a decrease in the heat storage. If integration in (10) is from the sample's surface to $X_t \to \infty$, the heat storage in the system remains constant for any α_{12} value after disconnection from the external source (Fig. 1b). This condition was used in testing the program. An analysis of the results of numerical investigation of the model on condition that the coefficients of interphase heat and mass exchange are constant (temperature-independent) has shown that the behavior of temperature is substantially dependent on the frequency and duration of the external action; this dependence is different for different values of the coefficients of heat exchange between the phases (particles and the matrix).

All the calculations presented below were carried out with the following values of the parameters [11]: c_{1s} = 0.692 J/(K·g), c_{2s} = 0.482 J/(K·g), $c_{1\text{liq}}$ = 0.693 J/(K·g), $c_{2\text{liq}}$ = 1.094 J/(K·g), ρ_{1s} = 4.92 g/cm^3, ρ_{2s} = 0.82 g/cm^3, $\rho_{1\text{liq}}$ = 4.238 g/cm^3, $\rho_{2\text{liq}}$ = 7.57 g/cm^3, k_1 = 0.21 W/(K·cm), k_2 = 0.295 W/(K·cm), R = 8.31 J/(mole·K), $Q_{\text{ph}1}$ = 387.9 J/g, $Q_{\text{ph}2}$ = 300 J/g, $\varepsilon = \varepsilon_1 = \varepsilon_2$ = 0.5, $T_0 = T_{\text{env}}$ = 300 K, $T_{\text{ph}2}$ = 1637 K, $E_{\text{a}1}$ = 79 J/mole, $E_{\text{a}2}$ = 163 J/mole, D_{10} = 3.18·10^{-7} cm^2/sec, D_{20} = 9·10^{-7} cm^2/sec, C_0 = 0.001, and σ = 5.67·10^{-12} W/(cm^2·K^4).

Figure 2 exemplifies the surface temperatures of particles and the matrix as functions of time under the action of a constant heat flux ($t_{\text{imp}} \to \infty$) and without allowance for the melting of the matrix ($Q_{\text{ph}2}$ = 0). The higher the coefficient of interphase heat exchange, the closer the particle and matrix temperatures. Under the conditions of intense heat exchange (curves 3), they are coincident, in practice. An analogous result is obtained for the pulse character of heating [2].

Allowance for the melting of the matrix in accordance with law (5) gives rise to additional features: high-melting particles, whose temperature can fail to attain their melting point, "feel" the melting of the matrix, which will be shown below in investigating the complete model. Changing α_{12} values in a certain manner, one can achieve the melting of the matrix without the melting of particles and to increase or decrease the width of the sample's warming-up zone [13, 14].

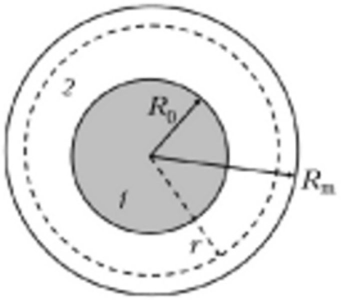

Fig. 3. On the formulation of the problem on determination of the heat-exchange coefficient: 1) particle; 2) matrix.

Determination of the Heat- and Mass-Exchange Coefficients on the Basis of Solution of a Conjugate Problem. The real value of the coefficients of interphase heat exchange is dependent on the character of the boundary between particles and the matrix, on the wettability of the particles with a molten matrix, etc. and can be different for different systems. Furthermore, the physical meaning of the coefficient is clear for the heat exchange between particles and fluid media. In this case the heat exchange is dependent on the character of flow. It turns out that the heat-exchange coefficient is far from being always constant. Calculation of the coefficient α_{12} in the models of the mechanics of heterogeneous media [15] presents a special problem. It is dependent on many factors, including the particle size and the temperature at the boundary $\alpha_{12} = \alpha(R_0, T)$. To determine the heat-exchange coefficient we must solve an individual problem. Analogous arguments can be given for special determination of the coefficient of interphase mass exchange [16] $\beta_{12} = \beta(R_0, T)$.

In the literature, there is no formula which gives such a relationship in the general case. Therefore, to determine the coefficients of interphase heat and mass exchange we have solved a special problem appropriate for our case, i.e., that of ideal contact between a particle and the matrix, typical of the selected model system. A mathematical formulation of the problem corresponds to Fig. 3.

Clearly, knowing the volume fraction of particles η in the matrix, we can calculate the average fraction of the matrix material per individual particle of radius R_0. Then the effective or average radius of the matrix fraction R_m per particle will be a function of η and R_0. Let us consider, as the model problem, heat exchange between a particle of radius R_0 and the matrix with effective radius R_m on condition that $R_m >> R_0$; ideal thermal contact is assumed between the particle and the matrix (as has been said above, the contact between titanium-carbide particles and the nickel-chromium binder may be considered ideal), whereas the symmetry condition is assumed at the center. The condition of absence of heat sources and sinks is set at the external boundary of the matrix region accounted for by a particle.

A mathematical formulation of the conjugate problem in a spherical coordinate system has the form [17]

$$c_1\rho_1 \frac{\partial T_1}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k_1 \frac{\partial T_1}{\partial r} \right), \quad 0 < r < R_0 ; \tag{11}$$

$$c_2\rho_2 \frac{\partial T_2}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k_2 \frac{\partial T_2}{\partial r} \right), \quad R_0 < r < R_m ; \tag{12}$$

$$k_1 \left. \frac{\partial T_1}{\partial t} \right|_{r=R_0} = k_2 \left. \frac{\partial T_2}{\partial t} \right|_{r=R_0} , \quad T_1 \,|_{r=R_0} = T_2 \,|_{r=R_0} ; \tag{13}$$

$$r = 0 : \ \frac{\partial T_1}{\partial r} = 0 , \quad r = R_m , \ \frac{\partial T_2}{\partial r} = 0 ; \tag{14}$$

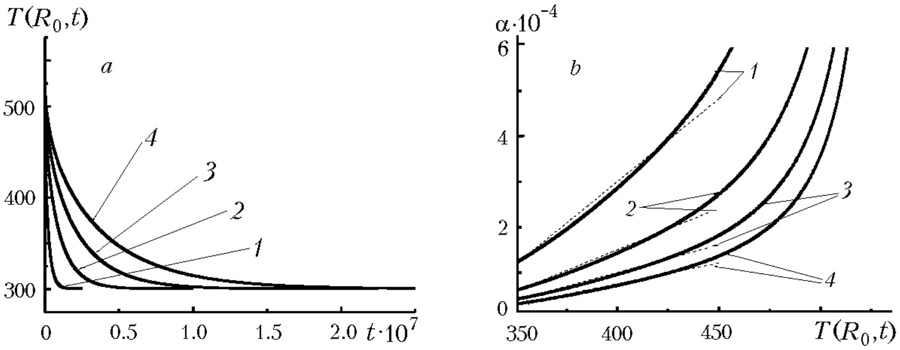

Fig. 4. Temperature at the phase boundary vs. time (a) and heat-exchange coefficient vs. this temperature (b) for different values of the particle radius (dotted curves, calculation from formula (16): 1) $R_0 = 50\cdot 10^{-4}$, 2) $100\cdot 10^{-4}$, 3) $150\cdot 10^{-4}$, and 4) $200\cdot 10^{-4}$ cm. α, W/(cm^2·K); t, sec.

$$t = 0: \ T_1 = T_s, \ T_2 = T_0. \tag{15}$$

Problem (11)–(15) has been solved by the same method as (1)–(9). The heat-exchange coefficient was computed from the formula [17]

$$\alpha = k_1 \left.\frac{\partial T_1}{\partial r}\right|_{r=R_0} \Big/ \left(T_0 - T_1 \big|_{r=R_0}\right). \tag{16}$$

Generally speaking, it is clear that the surface temperature of a particle decreases more slowly with increase in its size (Fig. 4a), i.e., such particles take longer to cool down. The temperature dependence of the heat-exchange coefficient for different particle sizes is shown in Fig. 4b. The heat-exchange coefficient calculated from formula (16) is exponentially dependent on time in the same manner as the temperature: it decreases with increase in the particle radius. The obtained curves can, in principle, be approximated by a single functional dependence by using different nonlinear functions (exponential function, a quadratic polynomial, a polynomial of the fourth degree, etc.), which is what we did. When (1)–(9) is used in the general model, a linear approximation containing less parameters and describing numerical results well in the temperature interval of interest turns out to be satisfactory. The fact that α_{12} values higher than 10^4 W/(cm^2·K) (Fig. 3, curves 3) may be considered as corresponding to ideal contact is also evidence in favor of the linear approximation. It turned out that the mass-exchange coefficient is in inverse proportion to the particle radius. As a result, we have obtained the approximation formula

$$\alpha = (A_0 + A_1 T(R_0, t)) \frac{R_*}{R_0}, \tag{17}$$

which describes the results obtained with an accuracy no worse than 2% in the temperature interval 350–450 K. In (17), A_0 and A_1 are the constants found by the least-squares method for $R_* = R_0$ (all the calculations were carried out for $R_* = 50$ μm, $A_0 = -145{,}791.11$, and $A_1 = 424.79$).

In accordance with [16], the expression for the mass-exchange coefficient in terms of the heat-exchange coefficient α has the form

$$\beta = \frac{\alpha}{k_1 / c_1 \rho_1} D_{10} \exp\left(\frac{E_{a1}}{RT_{ph2}}\right).$$

In a wider temperature interval (T = 300–500 K), it is the formula

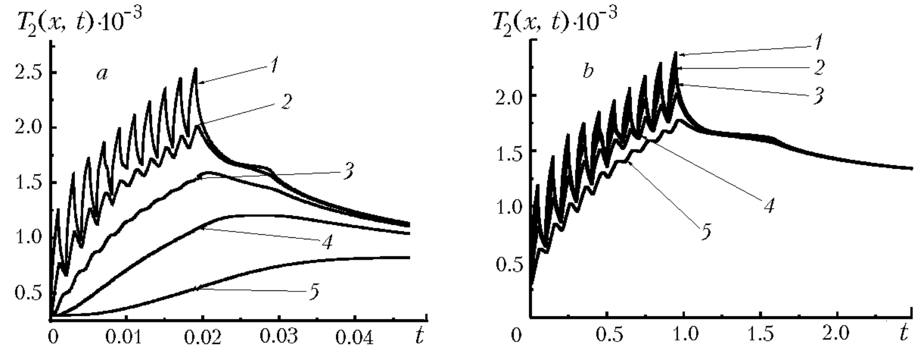

Fig. 5. Matrix temperature vs. time at different points of the sample [1) $x = 0$, 2) 0.01, 3) 0.025, 4) 0.05, and 5) 0.1 cm]: a) $q_0 = 5000$ W/cm^2, $t_{\text{imp}} = t_{\text{p}} = 10^{-3}$ sec, and $n = 10$; b) $q_0 = 1000$ W/cm^2, $t_{\text{imp}} = t_{\text{p}} = 5\cdot 10^{-2}$ sec, and $n = 10$. t, sec.

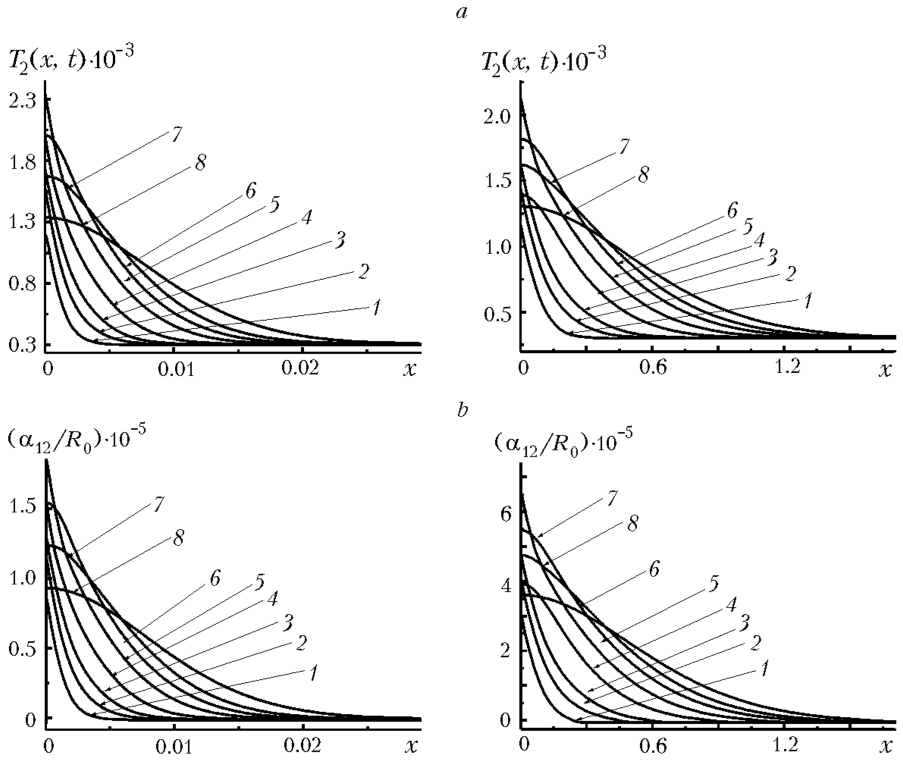

Fig. 6. Spatial distributions of the matrix temperature (a) and the heat-exchange coefficient (b) at different instants of time. On the left: $q_0 = 5000$ W/cm^2, $t_{\text{imp}} = t_{\text{p}} = 10^{-3}$ sec, $n = 10$, and $R_0 = 200\cdot 10^{-4}$ cm [1) $t = 0.001$, 2) 0.003, 3) 0.005, 4) 0.01, 5) 0.015, 6) 0.02, 7) 0.025, and 8) 0.035 sec]; on the right: $q_0 = 1000$ W/cm^2, $t_{\text{imp}} = t_{\text{p}} = 5\cdot 10^{-2}$ sec, $n = 10$, and $R_0 = 200\cdot 10^{-4}$ cm [1) $t = 0.05$, 2) 0.15, 3) 0.25, 4) 0.5, 5) 0.75, 6) 1, 7) 1.2, and 8) 1.5 sec]. x, cm.

$$\alpha = \exp\left(A_0 + A_1 T\,(R_0, t)\right)\frac{R_*}{R_0}\,, \quad A_0 = -\,119.98\,, \quad A_1 = 0.25$$

that is best suited to the coefficient of interphase heat exchange. To use the obtained relations $\alpha_{12} = \alpha(R_0, T)$ and $\beta_{12} = \beta(R_0, T)$ in problem (1)–(9) we must (from geometrical or dimensional considerations) divide (17) by R_0, i.e.,

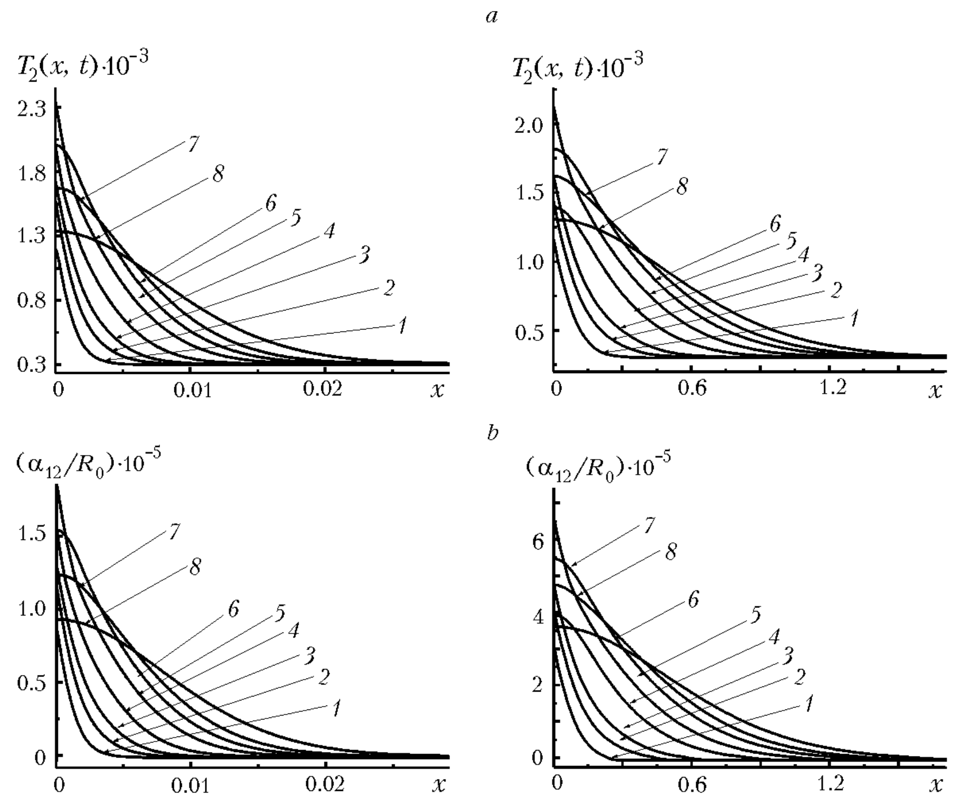

Fig. 7. Concentration of carbon in particles (a) and the matrix (b) vs. time at different points: 1) x = 0, 2) 0.01, 3) 0.025, 4) 0.05, and 5) 0.1 cm; n = 10 (q_0 = 5000 W/cm^2, $t_{\rm imp} = t_{\rm p} = 10^{-3}$ sec (A); q_0 = 1000 W/cm^2, $t_{\rm imp} = t_{\rm p} = 5\cdot 10^{-2}$ sec (B)).

$$\alpha_{12} = (A_0 + A_1 T(R_0, t)) \frac{R_*}{R_0^2}, \quad \beta_{12} = (A_0 + A_1 T(R_0, t)) \frac{R_*}{R_0^2} \frac{1}{k_1/(c_1\rho_1)} D_{10} \exp\left(\frac{E_{\rm a1}}{RT_{\rm ph2}}\right).$$

For determination of the temperature dependence of the heat- and mass-exchange coefficients in the case of nonideal contact (which can, e.g., be characterized by an additional thermal resistance dependent on the size of roughnesses), we have solved a similar problem but with a different condition at the phase boundary.

Thermal Problem with Variable Coefficients of Interphase Heat Exchange. Figure 5 gives the temperature at certain points of the treated sample as a function of time. Since the condition of ideal thermal contact was taken in calculations of the coefficient of interphase heat exchange, the particle temperature is virtually no different from the matrix temperature. It turned out that, despite the dependence of the effective heat-exchange coefficient on the parameter characterizing nonideal thermal contact, under nonstationary heat-exchange conditions, the nonideality of thermal contact between a particle and the matrix is rapidly smoothed out and has virtually no effect on the temperature distribution in particles and the matrix.

If we have $\Gamma = 1/(t_{\rm imp} + t_{\rm p})$ = 500 Hz, the pulse character of the external source is not pronounced at points at a distance $x >$ 0.025 cm from the surface (curves 3–5 in Fig. 5a), since the temperature field is smoothed out by heat conduction. The maximum temperature in the sample decreases with pulse-repetition frequency with preservation of the total energy "pumped" into the sample $q_0 n t_{\rm imp}$ = const, since heat has time to penetrate deep into the sample during the treatment because of which the depth of warning-up grows (Fig. 5b).

As the pulse frequency decreases (all other parameters being constant), the maximum temperature becomes lower and the warming-up zone increases, which is shown in Fig. 6a. The spatial temperature distributions correspond to the cessations of the first three pulses, the cessations of the fifth and tenth pauses, and to three instants of time in

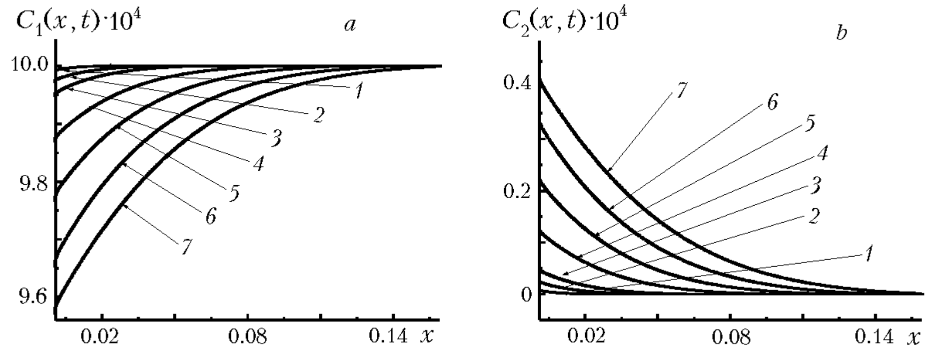

Fig. 8. Spatial distribution of carbon in particles (a) and the matrix (b) for q_0 = 5000 W/cm^2, $t_{\text{imp}} = t_{\text{p}} = 10^{-3}$ sec, n = 10, and $R_0 = 200\cdot10^{-4}$ cm at the instants of time: 1) t = 0.001, 2) 0.003; 3) 0.005, 4) 0.01, 5) 0.015, 6) 0.02, and 7) 0.025 sec.

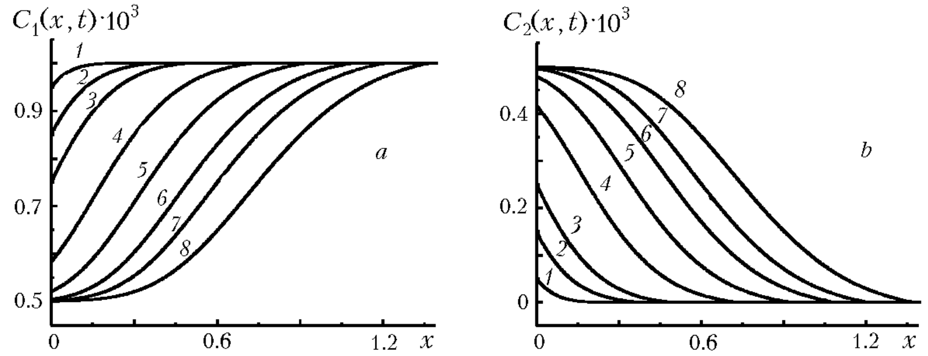

Fig. 9. Spatial distributions of carbon in particles (a) and the matrix (b) for q_0 = 1000 W/cm^2, $t_{\text{imp}} = t_{\text{p}} = 5\cdot10^{-2}$ sec, n = 10, and $R_0 = 200\cdot10^{-4}$ cm at the instants of time: 1) t = 0.05, 2) 0.15; 3) 0.25, 4) 0.5, 5) 0.75, 6) 1, 7) 1.2, and 8) 1.5 sec.

the step of subsequent cooling. Figure 6b gives the heat-exchange coefficient as a function of the space coordinate; it is natural that the coefficient acts in the same manner as the temperature. The form of the temperature dependence of the heat-exchange coefficient has no effect on the results of numerical investigation, since heat exchange in the system for $\alpha_{12} >$ 10,000 W/(cm$^2\cdot$K) is ideal. The particle temperature in this case is coincident with the matrix temperature; therefore, it is not shown in the figures. Consequently, in the case of ideal thermal contact between particles and the matrix we can restrict ourselves to the model [7] with effective thermophysical properties, even though the particle size exceeds the warming-up zone characteristic of one pulse.

Numerical Investigation of the Complete Model. The complete mathematical model assumed simultaneous solution of heat-conduction and diffusion equations. To solve the diffusion subproblem of problem (3)–(7) we have used the same method, i.e., the differential equations (3) and (4) were approximated by the implicit difference scheme by a four-point template, and the resulting system of algebraic equations was solved by the running method. In the numerical algorithm, we allowed for the difference of the processes of heat conduction and diffusion in the condensed phase in velocity and spatial scale. The diffusion problem was solved on a finer spatial grid; the results of solution of the thermal problem, "interpolated" to a finer grid, were used for calculation of the temperature-dependent parameters. In the calculations, we determined, in addition to the temperature, the concentration of the diffusing component (carbon in our case) in the matrix material and in the particle material at different instants of time.

The variation in the concentration of carbon in particles and in the matrix with time at different distances from the exterior surface is shown in Fig. 7. The larger the distance from the particles to the surface treated, the more slowly the carbon concentration varies in them because of the lower temperature; therefore, the concentration of carbon in the matrix decreases with increase in distance from the exterior surface. At $t_{imp} = t_p = 10^{-3}$ sec and $q_0 = 5000$ W/cm^2, the concentration of carbon in the particles changes only slightly as a result of the mass exchange with the matrix (Fig. 7A a), which leads to a low concentration of carbon in the matrix (Fig. 7A, b). The pulse character of heating is not pronounced in this case. At a distance larger than 0.1 cm, we can disregard the dissolution of particles. The material is not warmed up higher than 900 K at this distance (see Fig. 6). This is also seen from the spatial distributions (Figs. 8 and 9). In the case $t_{imp} = t_p = 5 \cdot 10^{-2}$ sec and $q_0 = 1000$ W/cm^2 the concentration of carbon in particles and the matrix at the same distance has time to be equalized over a period shorter than the duration of ten pulses (Fig. 7B); the pulse character of heating begins to have an effect on the concentration curves (Figs. 8 and 9). The curves of the time dependence of the concentration of carbon in particles and in the matrix have characteristic bends corresponding to the instants of pauses in the process of treatment. The zone of intense mass exchange increases with warming-up zone (Fig. 9). Such a "deep" redistribution of the concentrations can give rise to undesirable compounds and phases, once a certain value of the carbon concentration in the matrix has been attained, which has been allowed for in the general formulation of the problem but is not the subject of the present investigation.

Consequently, by changing the pulse-repetition frequency and the pulse duration and energy, we can control the depth of processing, which is consistent with experimental data [3, 7].

Conclusions. Study of the proposed model of pulse electron-beam processing of a heterogeneous material leads to results consistent with experimental data. Thus, it has been shown that the character of the temperature and concentration distribution in substances is substantially dependent on the parameters of interphase heat and mass exchange and the parameters of the external source. Varying the parameters of a pulse source, we can change the depth of processing which results in the redistribution of the components between particles and the matrix; in a real situation, this is related to the formation of transition diffusion layers and to the strengthening of the surface layer. This points to the adequacy of the model proposed for the observed regularities and to the possibility of using it further for description of the process of modification of the surface properties of a heterogeneous material. This, certainly, will require that more subtle processes resulting in the change in the structure and properties of surface layers be allowed for in the model. Thus, short pulse durations and high energies in a pulse, typical of processing conditions [3], produce other interesting effects disregarded in model (1)–(9), e.g., the destruction of particles because of thermal stresses due to the high temperature gradient. Internal stresses result from both diffusion and chemical reactions that can give rise to new compounds and phases with properties different from the properties of the starting substances. Allowance for the physicochemical transformations will inevitably lead to a complication of the model and to the necessity of determining the corresponding formally kinetic parameters theoretically or experimentally, which requires separate study.

Allowance for the diffusion of other components creates no fundamental difficulties from the mathematical viewpoint or in the context of formulation of the problem. Also, this inevitably involves the appearance of additional parameters (pre-exponents and activation energies) and the development of special numerical algorithms enabling one to investigate such related problems in a wide range of variation in the processing parameters. But even in such a simplified variant, the model proposed can be used for formulation and investigation of the problem of optimization of the process of surface treatment of a heterogeneous material.

NOTATION

$(c_i)_{eff}$, effective heats, J/(g·K); C_i and C_0, concentration and initial concentration of the alloy component, dimensionless parameter; D_i, diffusion coefficients, cm^2/sec; E_{ai}, activation energies, J/mole; k_i, thermal conductivities, W/(cm·K); q_0, maximum flux-power density, W/cm^2; q_t, heat storage in the system, J/cm^2; Q_j, heat release in the reaction with No. j, J/cm^3; Q_{phi}, heat of phase transition, J/mole; r, running radius, cm; R, universal gas constant, $R = 8.31$ J/(mole·K); R_0, particle radius, cm; R_m and R_*, matrix radius and scale radius respectively, cm; X_t, width of the warming-up zone, cm; x, coordinate, cm; t, time, sec; t_{imp} and t_p, pulse and pause time respectively, sec; T_0, initial matrix temperature, K; T_s, initial particle temperature, K; T_i, absolute temperature, K; T_{ph2}, melting temperature (point) of the matrix, K; w_j, rate of formation or loss of the component in the volume because of the chemical reactions, sec^{-1}; α_{12}, coefficient of interphase heat exchange, W/(cm^2·K); β_{12}, mass-exchange coefficient, sec^{-1}; δ, Dirac delta function; Γ, frequency, Hz; η, volume fraction of particles, dimensionless parameter; ε_i, emissivity, dimensionless parameter; $(\rho_i)_{eff}$, effective densities, g/cm^3; σ, Stefan–Boltzmann constant, W/(cm^2·K^4). Subscripts: $i = 1$, parameters of particles, $i = 2$, parameters of the matrix; a, activation; eff, effective; env, referring to the environment; imp, pulse; liq, liquid; m, matrix; p, pause; ph, phase; s, solid; t, thermal; j, No. of reaction (if they are allowed for).

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